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Effect of Magnetic Field on Coupling Parameter in Zeeman Laser

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Abstract—*The Zeeman laser theory subject to a uniform dc magnetic field at any angle to the maser axis was given in extension of a non-Zeeman treatment by Lamb. In this work, we have derived self-consistency equations in Zeeman laser using the complex conjugate terms which have been ignored in the original works of Lamb. One of the salient features of this calculation is that additional terms appear in self-consistency equations. In Zeeman laser theory, the relationship between the coupling parameters and magnetic fields around the cavity detuning values has been derived. It is observed that the detuning values of the coupling parameters have larger values for reasonably strong magnetic fields. This is one of the salient features of the Zeeman laser. The coupling decreases as the magnetic field strength is increased.*

Keywords*: Zeeman laser, coupling parameter, self-consistency equation, laser cavity.*

1. INTRODUCTION

The theory of Zeeman laser subject to a uniform dc magnetic field at any angle to the maser axis was given in extension of a non-Zeeman treatment by Lamb [1- 10]. The polarization modes can be expressed in terms of a "coupling" parameter C. This depends on the atomic angular momenta but decreases sharply as a function of magnetic fields. Lamb and his coworkers explained the various fields such as EM field equations, polarization of the medium, equation of motion, cavity anisotropy, transverse magnetic field, atomic decay rate, Lande's factor etc. in laser. The EM field is treated classically for a general state of polarization in a cavity with any desired degree of cavity anisotropy. We have modified self-consistency equations in Zeeman laser using the complex conjugate terms which have been ignored in the original works of Lamb [1, 2, 11]. In this calculation some additional terms are appeared in self-consistency equations.

2. METHOD & THEORITICAL ESTIMATION

The active medium consists of thermally moving atoms of varying isotopic abundance which have two electric states with arbitrary angular momenta. The self-consistency [11] requirement that a quasi-stationary field should be sustained by the induced polarization that leads to the equations which determine the amplitudes and frequencies of multimode oscillations as functions of the laser parameters.

A relationship between the coupling parameters and magnetic fields around the cavity detuning values has been derived in Zeeman laser theory. It has been shown that the upper parameter γ_a (decay from the sublevels) and γ_b (decay from the lower sublevels) as shown in **Fig. 1,** involving a particular pair of magnetic sublevels influence the coupling phenomenon.

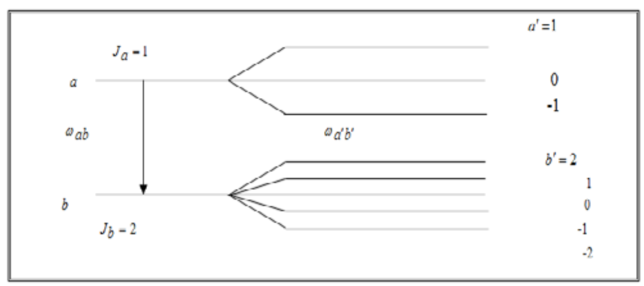

Fig. 1 Possible level scheme showing how levels 'a' and 'b' might be split into

$2J_a+1=3$ and $2J_b+1=5$ sublevels, respectively, by an applied dc magnetic field.

By using complex conjugate terms for electric field and polarization in Maxwell's equation for Zeeman laser and by equating the real and imaginary parts of finally derived equation separately to zero, we get the self-consistency equations as

$$\dot{E}_+ + \frac{1}{2}\nu[g_{++}E_+ + \mathrm{Im}\{g_{+-}E_- \exp(i\psi)\}] = -\frac{1}{2}\frac{\nu}{\varepsilon_0}\mathrm{Im}(P_+) \tag{1}$$

and

$$\left(2\upsilon_+ - \frac{\Omega_+}{\upsilon_+}\dot{\phi}_+ - \Omega_+\right)E_+ + i\frac{1}{2}\nu\,\mathrm{Re}[ig_{+-}E_- \exp(i\psi)] = -\frac{1}{2}\frac{\nu}{\varepsilon_0}\mathrm{Re}(P_+) \tag{2}$$

For diagonal losses, the self-consistency equations (1) and (2) reduce to

$$\dot{E}_{+} + \frac{1}{2}\frac{\upsilon_{+}}{Q_{+}} E_{+} = \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}\operatorname{Im}(P_{+}) \tag{3}$$

and

$$\left(2\upsilon_{+} - \frac{\Omega_{+}}{\upsilon_{+}}\dot{\phi}_{+} - \Omega_{+}\right) = -\frac{1}{2}\frac{\upsilon}{\varepsilon_0} E_{+}^{-1}\operatorname{Re}(P_{+}) \tag{4}$$

It may be inferred from equation (3) and (4) that in the absence of any active medium $P_{+} = 0$, we have

$$\dot{E}_{+} = -\frac{1}{2}\frac{\upsilon_{+}}{Q_{+}} E_{+} \quad \text{and} \quad \left(2\upsilon_{+} - \frac{\Omega_{+}}{\upsilon_{+}}\dot{\phi}_{+}\right) = \Omega_{+}$$

From equation (4), we have

$$\Rightarrow [\upsilon_{+} + \upsilon_{+} + \dot{\phi}_{+} - (\dot{\phi}_{+} + \frac{\Omega_{+}}{\upsilon_{+}}\dot{\phi}_{+})] = \Omega_{+} + \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1}\operatorname{Re}(P_{+}) - \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1}\operatorname{Re}(P_{+}) - \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1}\operatorname{Re}(P_{+}) \tag{5}$$

Thus it is observed that equation (5) consists of two parts,

$$\upsilon_{+} + \dot{\phi}_{+} = \Omega - \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1} \text{ Re part of } (P_{+}) \tag{6}$$

This is same as equation derived for real terms and the other part is

$$\left[\upsilon_{+} - \left(\dot{\phi}_{+} + \frac{\Omega_{+}}{\upsilon_{+}}\dot{\phi}_{+}\right)\right] = \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1}\operatorname{Re}(P_{+}) - \frac{1}{2}\frac{\upsilon_{+}}{\varepsilon_0}E_{+}^{-1}\operatorname{Re}(P_{+})$$

or

$$\upsilon_{+} - \dot{\phi}_{+} = \frac{\Omega_{+}}{\upsilon_{+}}\dot{\phi}_{+} \tag{7}$$

Thus equation (5) is equivalent to two equations. The first equation is the original equation of the semiclassical theory representing dispersion and the second equation also represents dispersion but with different form [3, 10]. It is reasonable to believe that the additional term apeared in equation (7) will be contributing in the determination of atomic decay rates and 'g' values for the laser atoms.

A complete atomic model consists of a system with $2J_a + 1$ upper state and $2J_b + 1$ for lower states; where J_a and J_b are the total angular momentum for the upper and lower state respectively as shown in **Fig. 1**. Zeeman splitting of the magnetic sublevels is defined by

$$\omega_{a'} = \omega_a + \mu_B H g_a \frac{a'}{\hbar}$$

Where, $\hbar\omega_a$ is the zero- field energy of the upper states, μ_B is the Bohr magnetron, H is the applied magnetic strength and g_a is the Lande's factor for the upper states. In calculating the polarization of atomic medium the additional terms are expected to influence the coupling parameter given by

$$c = \theta_{+-}\theta_{-+} / \beta_{+}\beta_{-} \tag{8}$$

The cross-saturation term can be written as:

$$\theta_{+-} = \theta_{-+} = \frac{1}{2}\frac{\gamma_a}{\gamma_{ab}}[\mathcal{I}(\delta) + \mathcal{I}(\omega_0 - \nu_0)]\mathrm{F}_3 + \frac{1}{2}\frac{\gamma_a}{\gamma_{ab}}[\mathcal{I}(2\delta) + \{\mathcal{I}(\delta)[1 - 2\delta^2(\gamma_a\gamma)^{-1}] + \mathcal{I}(\omega_{+\mathrm{b}} - \nu_{+})[1 - 2\delta(\omega_{+\mathrm{b}} - \nu_{+}(\gamma_a\gamma)^{-1}] \tag{9}$$

and the self-saturation term can be written as:

$$\beta_{+} = \beta_{-} = F_3 \sum_{a'}\sum_{b'} \delta_{a'b'\pm 1}\left(\frac{\wp_{a'b'}}{\wp}\right)^4 [1 + \mathcal{I}(\omega_{a'b'} - \nu_{\pm})] \tag{10}$$

Now, using equations (9) and (10) in equation (8), we may calculate the coupling parameters for different values of J = 2↔J = 2; J = 1↔J = 0 and J = 1↔J = 2 transition.

The coupling constant is related to the J values by the following equation.

$$c = \begin{Bmatrix} \left[\dfrac{(2J+3)(2J-1)}{2J^2+2J+1}\right]^2, & \mathbf{J} \leftrightarrow \mathbf{J} \\ \left[\dfrac{2J^2+4J+5}{6J^2+12J+5}\right]^2, & \mathbf{J} \leftrightarrow \mathbf{J+1} \end{Bmatrix} \tag{11}$$

3. SIMULATION

Using equations (9) and (10) in equation (7), we have calculated the coupling parameters for different values of J = 2↔J = 2; J = 1↔J = 0 and J = 1↔J = 2 transition.

The coupling constant is related to the J values by the equation (11).

We plot the graph in Microsoft excel platform representing the coupling parameter versus axial magnetic field at constant value of (J) using the calculated values. The responses are as shown in **Fig. 2** [axial magnetic field strength ranging from (0 to 2) Gauss and (0 to -2) Gauss and **Fig.3** [axial magnetic field strength ranging from (0 to 12) G and (0 to -12) G].

The parameters used in the calculation are, γ_a = 18 MHz, γ_b = 40 MHz, γ_{ab} = 29 MHz, *Ku*=1010 MHz, $\prod$ = 1.2, $g_a = g_b$ = 1.295, γ = 2π × 29 MHz, μ_B = 9.27×10^{-24} J / K, $\hbar$ = 1.027×10^{-34} Js, ε_0 = 8.85×10^{-12} c^2 /Nm^2

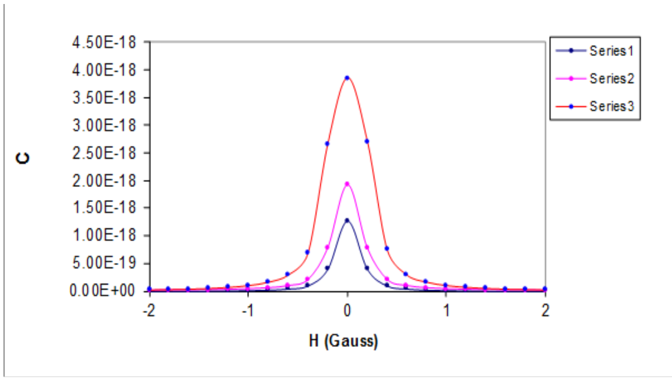

Fig. 2 Coupling Parameter C is plotted against axial magnetic field in gauss.

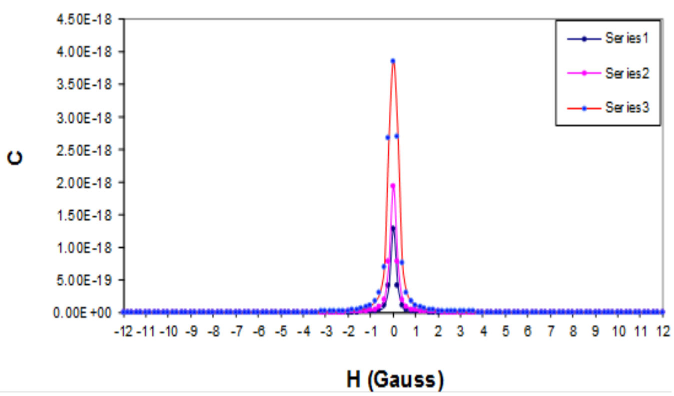

Fig. 3: Coupling parameter C is plotted against axial magnetic field in gauss.

4. RESULT & DISCUSSION

As may be observed from the **Fig. 2** and **Fig. 3,** when the magnetic fields are reasonably strong, the detuning values of the coupling parameters have larger values. The coupling parameters C1, C2 and C3 depend primarily on the magnetic fields around the detuning values. This is one of the salient features of the Zeeman laser. In general, the coupling decreases as the magnetic field strength is increased. For two running waves polarized in opposite direction and traveling in the same direction, try to interact with atoms in the same velocity range. This is in fact an important consequence of the Zeeman laser theory originally formulated by Lamb.

5. CONCLUSION

The coupling decreases as the magnetic field strength is increased. It may be inferred from the responses that there are three regions of coupling; strong coupling region is indicated by C_3, weak coupling region is indicated by $C_{1,}$ and neutral coupling is given by C_2 and in general $C_1 \rangle C_2 \rangle C_3$. It depends on J values and magnetic sublevels. It does not influence the basic theory it indicates that decay parameters play a major role in Zeeman laser.

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